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On the Geometric Convergence of Fractional Invariants in Non-Euclidean Topologies

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Abstract. *This paper establishes an analytical framework for calculating the geometric convergence rates of fractional invariant measures on complete Riemannian manifolds. By extending classical localization techniques to localized fractional laplacians, we prove that the structural stability of topological entropy remains invariant under bounded metric perturbations. Our findings generalize several foundational theorems in non-Euclidean dynamics and provide new algorithmic lower bounds for multi-scale numerical integration schemes.*

1 Introduction

Let $(\mathcal{M}, g)$ be a complete, smooth Riemannian manifold with non-positive sectional curvature. The study of invariant measures under the action of fractional diffusion operators has seen significant development. In this paper, we address the open problem regarding structural stability limits when the metric g undergoes local variations.

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2 Core Theoretical Results

We begin by establishing the necessary analytical bound requirements to verify that your class file handles structural mathematical formulas alongside standard mathematical theorem counters cleanly.

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Definition 2.1. *A fractional invariant measure μ is said to be geometrically regular if there exists a localized tracking constant $\Lambda > 0$ such that for all test functions $\phi \in C_c^\infty(\mathcal{M})$:*

$$\int_{\mathcal{M}} (-\Delta)^\alpha \phi \, d\mu \leq \Lambda \|\phi\|_{L^2(\mathcal{M})}. \tag{1}$$

With our basic definitions established, we present the principal limiting convergence theorem below.

Theorem 2.2 (Main Convergence Property)**.** *Let μ_n be a sequence of geometrically regular fractional invariant measures on $(\mathcal{M}, g)$. If the structural tracking capacity satisfies $\lim_{n\to\infty} \Lambda_n = \Lambda_\infty$, then μ_n converges weakly to a limiting measure μ_∞ satisfying the metric condition:*

$$|\mu_n(E) - \mu_\infty(E)| = \mathcal{O}\left(n^{-\frac{2\alpha}{d}}\right). \tag{2}$$

Proof. The proof proceeds via standard localization techniques. Let $\Omega \subset \mathcal{M}$ be a compact subset. By applying the fractional Sobolev embedding theorem directly to our kernel matrix, we deduce that the local trace operator is completely bounded. The remaining convergence constraints follow immediately from standard density arguments. □

Lemma 2.3. *Under the assumptions of Theorem 2.2, any bounded perturbation of the background manifold metric g preserves weak compactness properties.*

Remark 2.4. Note that if $\alpha \to 1$, the convergence rate smoothly degenerates back into the classical local operator bounds established by classical diffusion models.

3 Numerical Verification and Benchmarking

To verify how multi-column floats or layout matrices interact with your document text block width (6.1 in), numerical test suites were compiled using local cluster instances. The resulting text flows cleanly around the mathematical frameworks.

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